Vorticity Reynolds Number and its Relation to Momentum Reynolds Number in Wedge and Corner Flow

Jonathan B. Adams
University of Kentucky College of Engineering, Paducah, Kentucky, 42001

With increasing importance upon CFD in today's engineering community, a greater understanding of Boundary-layer theory is a necessity. In this paper, a newly found relation for Falkner-Skan wedge flows will be presented. This relation allows individuals to discover a constant that will relate the Vorticty Reynolds Number to the Momentum Reynolds Number for certain wedge angles and flow conditions. Further steps will be outlined so that others may expand upon this research and transform these relations into functions of pressure gradients under the same flow conditions. These newly found relations will generate time saving benefits in CFD analysis.

Nomenclature

A	=	amplitude of oscillation
g_x	=	acceleration due to gravity in the x direction
g_y	=	acceleration due to gravity in the y direction
g_z	=	acceleration due to gravity in the z direction
$g(x)$	=	dimensionless scale factor
H	=	shape factor, (δ_1/δ_2)
K	=	acceleration parameter
L	=	length
m	=	constant to replace beta
Re_x	=	Reynolds number, $(\rho L U_{ref}/\mu)$
Re_ω	=	vorticity Reynolds number, $(\omega d^2/\nu)$
t	=	time
U_∞	=	free stream velocity
u	=	velocity component in the x-direction
V	=	velocity magnitude
v	=	velocity component in the y-direction
w	=	velocity component in the z-direction
β	=	wedge angle
δ	=	a number that is approximately zero
δ_1	=	displacement thickness
δ_2	=	momentum thickness
φ	=	Adams number
η	=	dimensionless coordinate
μ	=	dynamic viscosity
ν	=	kinematic viscosity
ρ	=	density
ψ	=	stream function

I. Introduction

THE concept of Boundary-layer theory has been the center of much research and study since Ludwig Prandtl first proposed the idea over 100 years ago. The modern engineering world has become dependent upon the knowledge developed from Boundary-layer research and exploration. Since CFD software packages have become commercially available, the general public has had access to the power of these solution techniques. Nevertheless, there are always areas for improvement.

The original idea for this research came from an article by Menter in 2004 in which Menter described an approximation for a relation that describes the vorticity Reynolds number at its maximum value as a function of the Reynolds number of the momentum thickness, inside the boundary layer for Falkner-Skan flows at zero incident in the form of a simple constant. This relation involved certain assumptions and flow conditions which will be presented. The findings that were presented in the publication by Menter begged the question that led to this research. Given the fact that Falkner-Skan flows at zero incident do not harbor any pressure gradients in their equations or physical makeup, would it be possible to discover constants that could relate vorticity Reynolds number at its maximum value to the Reynolds number of the momentum thickness for any given Falkner-Skan flow? Further, knowing that the only variance in different Falkner-Skan flow conditions is the wedge angle, would it be possible to present the constants as a function of the wedge angle? Thus, the search for such a constant commenced and led to the findings presented in this publication. However, for an individual to develop a complete understanding of the physical significance of these findings, and to continue future research in this topic, an introductory background will be given in the following derivations in so that all assumptions and derivations may be understood.

II. Derivation and Assumptions

Boundary-layer theory is the concept that there is a difference in thickness where the velocity of the flow field closest to the body will match the free stream velocity of the flow field in the y-direction as one travels along the x-direction of any given body within a flow field, which is directed in the x-direction in that flow field, in a conventional two dimensional Cartesian coordinate systems. There are some fundamental concepts that must be presented before one can fully appreciate the work of Prandtl, Blasius, Falkner, and Skan. Two of the more popular solutions to the boundary layer problem were presented by Blasius, Falkner, and Skan: however, important contributions by Prandtl and others cannot be ignored. Since the work of Menter is based on the Blasius solution, its derivation, as well as the derivation of the Falkner-Skan equation, will be presented in the following, in addition to all the assumptions that allowed for the simplifications of highly complicated partial differential equations. We will begin our derivation by starting at the Navier-Stokes equations. They can be stated as follows for incompressible Newtonian fluids:

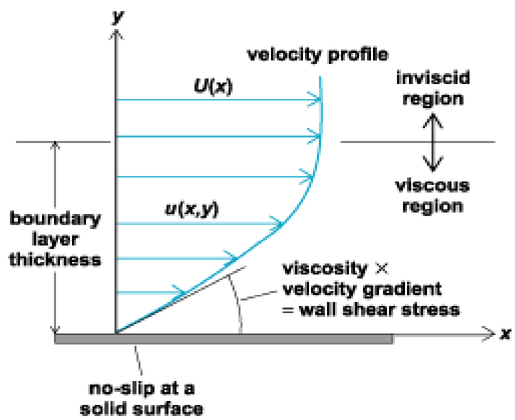


Figure 1. Boundary Layer Along a Flat Plate, ***shows the various aspects and anatomy of the boundary layer development along the flat plate.*** <u>**www.answers.com**</u>

The Navier-Stokes equation in the x-direction is:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

The Navier-Stokes equation in the y-direction is:

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{2}$$

The Navier-Stokes equation in the z-direction is:

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial w}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial w^2}\right) \tag{3}$$

When these three equations are combined with the conservation of mass equation which reads:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4}$$

it provides four equations and four unknowns (u,v,w,p). One important aspect of the Navier-Stokes equations is that they are nonlinear, second-order, partial differential equations. Now the derivation of the boundary layer equations for flow along a flat plate at zero incidence can begin. First, a list of the assumptions that need to be made should be presented. One assumption is the fact that the focus of this research deals only with two dimensional fluid flow, thus Eq. 3 can be neglected.

There are two important points to consider in boundary layer theory:

1) A very thin layer in the immediate neighborhood of the body in which the velocity gradient normal to the wall, $\partial u/\partial y$ is very large (boundary layer). In this region the very small viscosity μ of the fluid exerts an essential influence in so far as the shearing stress $\tau=\mu(\partial u/\partial y)$ may assume large values.
2) In the remaining region, no such large velocity gradients occur and the influence of viscosity is unimportant. In this region the flow is frictionless and potential. (Schlichting 1979)

It should be noted that the boundary layer becomes larger as the dynamic viscosity increases. In addition, the Reynolds number increases as the boundary layer decreases. Thus, it can be stated that the boundary-layer thickness is proportional to the square root of the kinematic viscosity (Schlichting 1979):

$$\delta \sim \sqrt{\nu} \tag{5}$$

where δ is denoted as the boundary layer thickness and ν represents the kinematic viscosity. This holds true as long as it is assumed that the boundary layer thickness is very, very large in respect to the characteristic length of the body in question. In this way, the Boundary-layer equations are asymptotic due to the very large Reynolds number.

The Navier-Stokes equations can now be presented in a dimensionless form, and estimates of the order of magnitudes present in the equations may be made by observing the following assumptions.

1) Referring all velocities to the free stream velocity
2) Referring all dimensions to a characteristic length of body which is selected so that $\partial u/\partial x$ does not exceed unity.
3) Making pressure dimensionless by using the term ρV^2
4) Referring to time as L/V
5) Using the definition of the Reynolds number, $\mathrm{Re}=\frac{VL\rho}{\mu}=\frac{VL}{\nu}$
6) Using the Equation of Continuity.

By adhearing to these assumptions, the following equations may be presented, along with the estimated orders of magnitude. For the proceeding equations, δ is considered to be much, much less than unity.

The simplified flow equation for the x-direction:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{6}$$

$$1 \qquad 1\ \ 1 \quad \delta\,\frac{1}{\delta} \qquad\qquad \delta^2 \quad 1 \quad \frac{1}{\delta^2}$$

The simplified flow equation for the y-direction:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{7}$$

$$\delta \quad 1 \quad \delta \quad \delta \quad 1 \qquad \delta^2 \quad \delta \quad \frac{1}{\delta}$$

By assuming a no slip condition between the working fluid and the wall of the body in question, the boundary conditions consist of zero flow in the x and y directions at y equal to zero, and u=U for $y=\infty$.

For as long as the boundary layer thickness is believed to still remain much, much less than unity, the following assumptions can be made. Referring back to Eq. 7, and considering the fact that there is no large velocity gradient present, the viscous terms in the equation drop out for large Reynolds number values, providing the subsequent equation:

$$\frac{\partial U}{\partial t}+U\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{8}$$

Noting that the solutions that are of importance for the relations and flow conditions of interest are steady state, the time dependent term disappears providing the following:

$$U\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{dp}{dx}. \tag{9}$$

Using Bernoulli's equation, Eq. 9 can be written as

$$p+\frac{1}{2}\rho U^2=const. \tag{10}$$

Now the simplified flow equations consist of Eq. 4 and the next equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}. \tag{11}$$

Eq. 4 and Eq.10 are know as Prandtl's Boundary Layer equations and are the basis of both the Blasuis and Falkner-Skan solutions. These equations can be further reduced by considering steady state conditions. In doing this, Eq. 11 becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+v\frac{\partial^2 u}{\partial y^2}. \tag{12}$$

In order to arrive at the Blasius solution, a few more steps must be taken. If we allow the leading edge of the plate to be at x=0, set the plate parallel to the x axis, and assume potential flow is constant, the pressure gradient in the x direction can be considered zero. This step further reduces Eq. 12 so that it may be expressed as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} \tag{13}$$

with boundary conditions existing at $u = v = 0$ at $y = 0$ and $u=U_\infty$ at $y = \infty$. Since these solutions will lead to the use of numerical techniques, it is convenient to introduce a dimensionless coordinate system so that the flow along the body can be located. This can be achieved by the following:

$$\eta \propto y/\delta \rightarrow \delta \propto \sqrt{\frac{vx}{U_\infty}} \rightarrow \eta = y\sqrt{\frac{U_\infty}{vx}}. \tag{14}$$

This new term can now be introduced into the stream function as:

$$\psi(x,y) = \sqrt{\nu x U_\infty}\, f(\eta)\,. \tag{15}$$

Velocity components become

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = U_\infty f'(\eta) \tag{16}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f' - f)\,. \tag{17}$$

By inserting these velocity components back into Eq.14, one is able to obtain the following equation

$$-\frac{U_\infty^2}{2x}\eta f' f'' + \frac{U_\infty^2}{2x}(\eta f' - f) f'' = \nu \frac{U_\infty^2}{x\nu} f'''\,. \tag{18}$$

If terms are expanded and canceled, the Blasius equation is revealed, and can be presented as:

$$ff'' + 2f''' = 0 \tag{19}$$

with boundary conditions at $f = 0$ and $f' = 0$ at $\eta = 0$, also $f' = 1$ at $\eta = \infty$.

The Blasius equation is a very powerful tool that gives engineers the ability to study Newtonian fluid behavior with an exact solution of the Navier-Stokes equations. Many concepts were generated from its use through the past, and in addition, it inspired others to continue research in this area. Since the Blasius equation only describes flow over a flate plate a zero incidence, it is impossible to understand the affects of pressure gradients upon the flow field when using the Blasius equation. Thus, it became necessary to create another equation that would be able to deal with angled flow conditions and their inherent pressure gradients. This was first done by Falkner and Skan. The equation that was the result of their work may be viewed below along with its derivation. By starting at the Prandtl's Boundary Layer equations and introducing the use of the stream function the following may be presented

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U\frac{dU}{dx} + \nu\frac{\partial^2 \psi}{\partial y^2} \tag{20}$$

with boundary conditions existing where $\frac{\partial \psi}{\partial x} = 0 = \frac{\partial \psi}{\partial y}$ at $y = 0$ and $\frac{\partial \psi}{\partial y} = U$ at $y = \infty$.

By reducing all lengths with the use of L as the characteristic length and by referencing all velocities to the potential flow velocity U_∞, the Reynolds number can be described as follows:

$$\mathrm{Re} = \frac{U_\infty L}{\nu} \tag{21}$$

Needing to make the y-coordinate dimensionless, everything is scaled by a factor of g(x) where

$$\zeta = \frac{x}{L}, \quad \eta = \frac{y\sqrt{\mathrm{Re}}}{Lg(x)} \tag{22}$$

Then the stream function can be made dimensionless by a simple substitution:

$$f(\zeta,\eta)=\frac{\psi(x,y)\sqrt{\mathrm{Re}}}{LU(x)g(x)} \tag{23}$$

Thus, the velocity components become,

$$U=\frac{\partial\psi}{\partial y}=U\frac{\partial f}{\partial\eta}=Uf',$$
$$-\sqrt{\mathrm{Re}\bullet}v=\sqrt{\mathrm{Re}}\,\frac{\partial\psi}{\partial x}=Lf\,\frac{d}{dx}(Ug)+Ug\left(\frac{\partial f}{\partial\zeta}-L\frac{g'}{g}\eta f'\right) \tag{24}$$

where *f'* is in respect to η, *x* is in respect to *g'*. By introducing the dimensionless variables presented above, the following expression can be obtained:

$$f'''+\alpha ff''+\beta(1-f'^2)=\frac{U}{U_\infty}g^2\left(f'\frac{\partial f'}{\partial\zeta}-f''\frac{\partial f}{\partial\zeta}\right) \tag{25}$$

where α and β are contractions for the following:

$$\alpha=\frac{Lg}{U_\infty}\frac{d}{dx}(Ug)\qquad \beta=\frac{L}{U\infty}g^2U' \tag{26}$$

Also, when the velocity $U'=dU/dx$, the boundary conditions become *f* = 0 and *f'* = 0 at $\eta=0$, also *f'* = 1 at $\eta=\infty$. Similar solutions exist where *f* and *f'* do not depend upon ξ, while α and β must remain independent of all values of x and remain constant, thus producing the Falkner-Skan equation, which can be stated as follows:

$$f'''+\alpha ff'+\beta(1-f'^2)=0 \tag{27}$$

where the boundary conditions are *f* = 0 and *f'* = 0 at $\eta=0$, also *f'* = 1 at $\eta=\infty$.

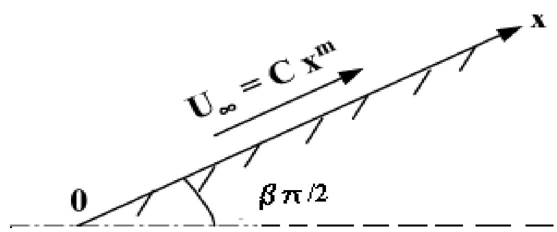


Figure 2. Falkner-Skan Wedge Flow, ***which shows the relationship between the wedge angle β and the geometry of the Falkner-Skan Flow. Above is a schematic of the geometry for β values between 0 and 2.***

Having knowledge of the wedge angle and its ramifications is pertinent to one's understanding of Falkner-Skan flow. The two figures that are presented here show the difference in the geometry and how the wedge angle describes that geometry. For β values between -0.2 and 0, the Falkner-Skan equations describe what is known as corner flow. In contrast, for β values between 0 and 2, the geometry and flow conditions described by the Falkner-Skan equations are known as wedge flow. It is important to note that for

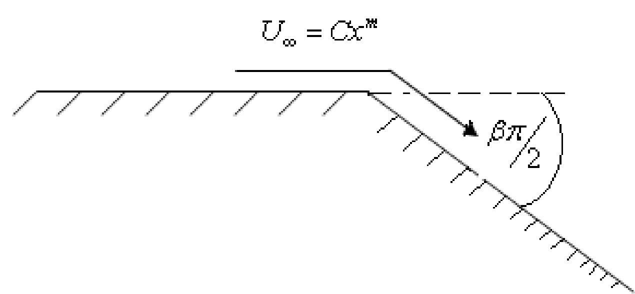


Figure 3. Falkner-Skan Corner Flow, ***which shows the relationship between the wedge angle β and the geometry of the Falkner-Skan flow. Above is pictured the geometry for β values between -0.2 and 0.***

wedge flows consisting of flat plates at zero incidence, all the way up to plates that are positioned at 90º, (completely orthogonal to the free stream velocity), Beta values will span from 0 to 1. This was to be the focus of this research, along with the entire corner flow region. Also, it is important to not lose sight of the fact that when the free stream velocity encounters an orthogonal wall, stagnation pressures are present and are equal to the measured static pressure.

III. Previous Findings

The relation of the vorticity Reynolds number to the Reynolds number of the momentum thickness for the Blasuis solution was first described by Menter in 2002 where the following equation was presented:

$$\mathrm{Re}_{\omega}(x,y)_{\max} \approx 2.193\,\mathrm{Re}_{\theta}(x). \tag{28}$$

Before the great in depth study could commence on finding a function which could produce a constant for any Falkner-Skan flow, it was important to demonstrate that the previous equation first described by Menter could be reproduced with available software and numerical techniques. Noting the boundary conditions of the Blasuis equation, and the output variables of the computer software to be used, further derivation was needed in order to produce the equation that was presented above. Since both the Blasuis equation and the Falkner-Skan equation differ by only one constant β (wedge angle), and given the fact that the numerical techniques used would present *f*, *f'*, and *f''* values for every η value along the plate for both flow conditions, it was first necessary for more derivation in order to produce the equations that would allow the above relation to be written.

IV. Current Findings

For this research, a computer program knows as MATLAB 7.0.4 was used to produce the numerical techniques described in this paper. MATLAB has a variety of ordinary differential equation solvers built within its programs. The one that was used for this project was labeled ode45, this version ode45 was used because it implemented forth and fifth-order Runge-Kutta methods (Palm III 2005). For MATLAB to use these solution techniques properly it is imperative that the differential equation that is to be solved, be broken up in to state variable form. Since the nature of the Falkner-Skan at zero incidence describes the same flow conditions as the Blasius equation, the equations them selves differ only by a factor of 2, while the constant that relates the vorticity Reynolds number to the Reynolds number of the momentum thickness by a factor of the square root of 2. From this point on, the constant that relates the vorticity Reynolds number to the Reynolds number of the momentum thickness shall be denoted as φ and called the Adams number.

The derivation that was needed involved just few steps. There had to be a mathematical way of describing the vorticity Reynolds number and the Reynolds number of the momentum thickness in terms of the output variables that would be presented by the numerical solutions produced by the computer software. In order to do this, the following steps were necessary. First, it is important to define some terms, in addition to what the output variables actually mean.

$$\frac{df}{d\eta} = f' = \frac{u}{U_{\infty}}\ , \quad f'' = \frac{\partial u}{\partial y} = \omega\ , \qquad \mathrm{Re}_{\omega} = \frac{\omega d^{2}\rho}{\mu} = \frac{\omega d^{2}}{\nu} \tag{29}$$

Furthermore, if the definition of the dimensionless coordinate is manipulated, it can be written as follows in a differential form with respect to y:

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{U_{\infty}}{\nu x}}\ . \tag{30}$$

Noting these relationships and definitions, the equation that defines the vorticity Reynolds number can be written as

$$\mathrm{Re}_{\omega} = \frac{\frac{du}{dy} d^2}{\nu} = \frac{U_\infty f'' \frac{\eta^2}{\frac{U_\infty}{\nu x}}}{\nu} . \tag{31}$$

By expanding and rewriting these terms, Eq. 31 condenses to the following:

$$\frac{\mathrm{Re}_{\infty}}{\sqrt{\mathrm{Re}_x}} = \eta^2 f'' \tag{32}$$

Realizing that the definition of a ratio for momentum thickness to length of a flat plate can be written as:

$$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \tag{32}$$

Further, by recognizing the definition of the Reynolds number of the momentum thickness as being,

$$\mathrm{Re}_\theta = \frac{U_\infty \theta}{\nu} = 0.644 \bullet \sqrt{\mathrm{Re}_x} \tag{33}$$

Considering the fact that the number 0.664 represents the momentum thickness of a flat plate at zero incidence, this number should be replaced with the definition of momentum thickness in order to describe this parameter at any angle. The equation for momentum thickness is as follows:

$$\delta_2 = \sqrt{\frac{2}{m+1}} \int_0^{\eta \to \infty} f'(1 - f') d\eta \tag{34}$$

For completeness, the equation for displacement thickness is given as:

$$\delta_1 = \sqrt{\frac{2}{m+1}} \int_0^{\eta \to \infty} (1 - f') d\eta \tag{35}$$

From Menter’s work, and Eq. 28, it can be seen by inspection that the constant will be in the form of Eq. 36 for each Falkner-Skan flow solution. Equation 36 can be shown below:

$$\mathrm{Re}_{\omega(\max)} = \varphi \sqrt{\mathrm{Re}_x} \tag{36}$$

By combining Eq. 36, 33, and 32, an expression that describes the constant in terms of the output variables can be obtained and written as follows:

$$\mathrm{Re}_{\omega(\max)} = \frac{\eta^2 f''}{\delta_2} \mathrm{Re}_\theta \tag{37}$$

Thus, it can be shown that the Adams number can be written as:

$$\varphi = \frac{\eta^2 f''}{\delta_2} \tag{38}$$

In addition to the parameters listed above, another commonly used variable is known as the shape factor which may be expressed as:

$$H = \frac{\delta_1}{\delta_2} \tag{39}$$

Also, it is sometimes useful to express the Falkner-Skan wedge angle *β* as *m*, an equation describing this relation can be written as follows:

$$m = \frac{2m}{m+1} \tag{40}$$

In order to verify that all of the computer code was working properly and that there were no errors in the derivations described above, the flat plate was studied first. It's important to note that in order to match *φ* to the constant that was described in Menter's work, that the Adams number would have to be multiplied by a factor of $\sqrt{2}$. Using four hundred *η* and the trapezoidal rule to calculate the displacement thickness and the momentum thickness, it was found for a flat plate of zero indecence, that the Adams number is equal to 1.54712. If multiplied by the proper factor, the constant becomes approximately equal to 2.1879. This number is close to Menter's number of 2.193. It is theorized that discrepancy can be accounted for by the difference in numerical techniques used to solve for the different values, in addition, for all of the numerical solutions described here, sixteen decimal places were used for the numerical code and calculations. An example of a mfile used in the MATLAB code and an Excel spreadsheet may be viewed in the appendix. Below, a table of the results generated by the computer code and the above calculations can be seen.

Table 1. Results of the Flow Field for Falkner-Skan Wedge and Corner Flows, ***showing the displacement thickness, momentum thickness, shape factor, and the Adams number for different β values of Falkner-Skan flows***.

b	*m*	Displacement Thickness, δ_1	Momentum Thickness, δ_2	*H*	f
-0.198838	-0.090428672	3.497865929	0.868131431	4.029189364	3.217458057
-0.18	-0.082568807	2.762956629	0.838162359	3.296445609	2.352240898
-0.15	-0.069767442	2.414806675	0.799373211	3.020875159	2.032860859
-0.1	-0.047619048	2.090660465	0.746361257	2.801137447	1.781167109
-0.05	-0.024390244	1.879092427	0.702249041	2.675820566	1.640079196
0	0	1.72092221	0.664145138	2.591183933	1.547127374
0.1	0.052631579	1.489354293	0.600314444	2.48095695	1.432575029
0.2	0.111111111	1.32069933	0.547821738	2.410819503	1.368339591
0.3	0.176470588	1.18809028	0.503055945	2.361745831	1.33299959
0.4	0.25	1.078630009	0.463832461	2.32547331	1.318571802
0.5	0.333333333	0.985495558	0.428997551	2.297205556	1.315472379
0.6	0.428571429	0.904060818	0.397449048	2.274658404	1.323256861
0.7	0.538461538	0.831405622	0.368494968	2.256219742	1.340688901
0.8	0.666666667	0.765464046	0.341594283	2.24085731	1.367144432
0.9	0.818181818	0.704715964	0.316320992	2.227850766	1.403070019
1	1	0.648007031	0.292340569	2.216616849	1.447406589

With all of these factors and variables in place, relationships between the various parameters could be developed and stated. Noting that there is a definite trend between the wedge angle *β*, and the Adams number *φ*, this relationship could be plotted and an equation could be stated that would give *φ* as a function of the wedge angle *β*. An equation that relates the Adams number to the wedge angle can be approximated by a 4th order polynomial for β values between 0 and 1 as:

$$\varphi = 0.9525\beta^4 - 2.4937\beta^3 + 2.799\beta^2 - 1.3547\beta + 1.5456 \tag{41}$$

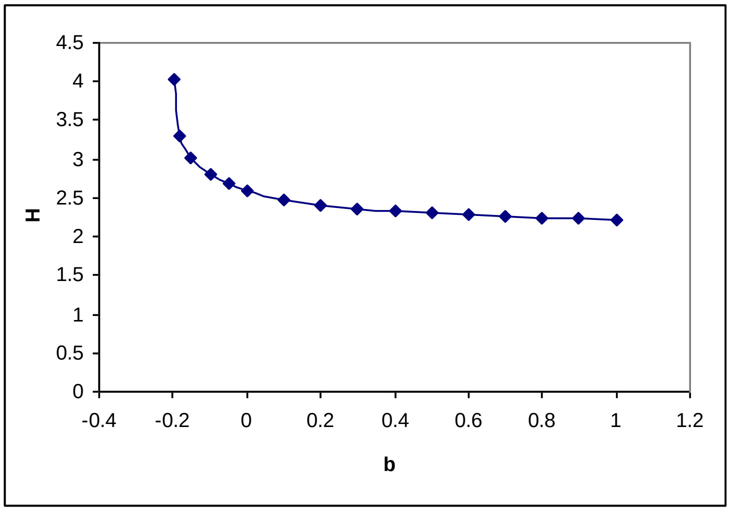


Figure 5. Shape Factor as a Function of Wedge Angle.

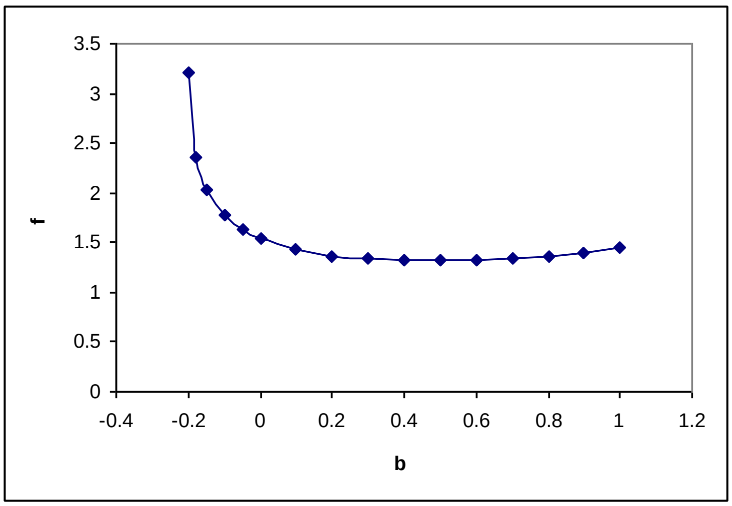


Figure 4. The Adams Number as a Function of the Wedge Angle for Falkner-Skan Flows.

Obviously, Eq. 41 does not account for wedge angles greater than 90º, or for corner flow; however, it is still useful and can be of much aid when preparing CFD code for transitional flow models. Of course, shape factor is commonly used when observing momentum thickness and its effects upon a flow field. A plot of shape factor as a function of wedge angle can also be viewed to the left.

In order to relate the Adams number to the corresponding wedge angles within Falkner-Skan corner flow, one must have a rational equation that includes an asymptote at -0.2. That may take some time; nevertheless, the benefit of such an equation could be well worth the effort.

V. Conclusion

The relationships that have been developed in this research will have real and lasting affect upon modern Computational Fluid Dynamics, and the associated analysis that is necessary to push the envelope of technology. Many important aspects were learned in the course of this research. One of which is the necessity of extreme precision when working with numerical techniques. In addition, a great deal about the general aspects of Boundary-layer theory were explored and expanded upon. Falkner-Skan is an important aspect in the history of Boundary-layer, and a good understanding of the mechanics involved with the concept of Falkner-Skan flows is extremely important.

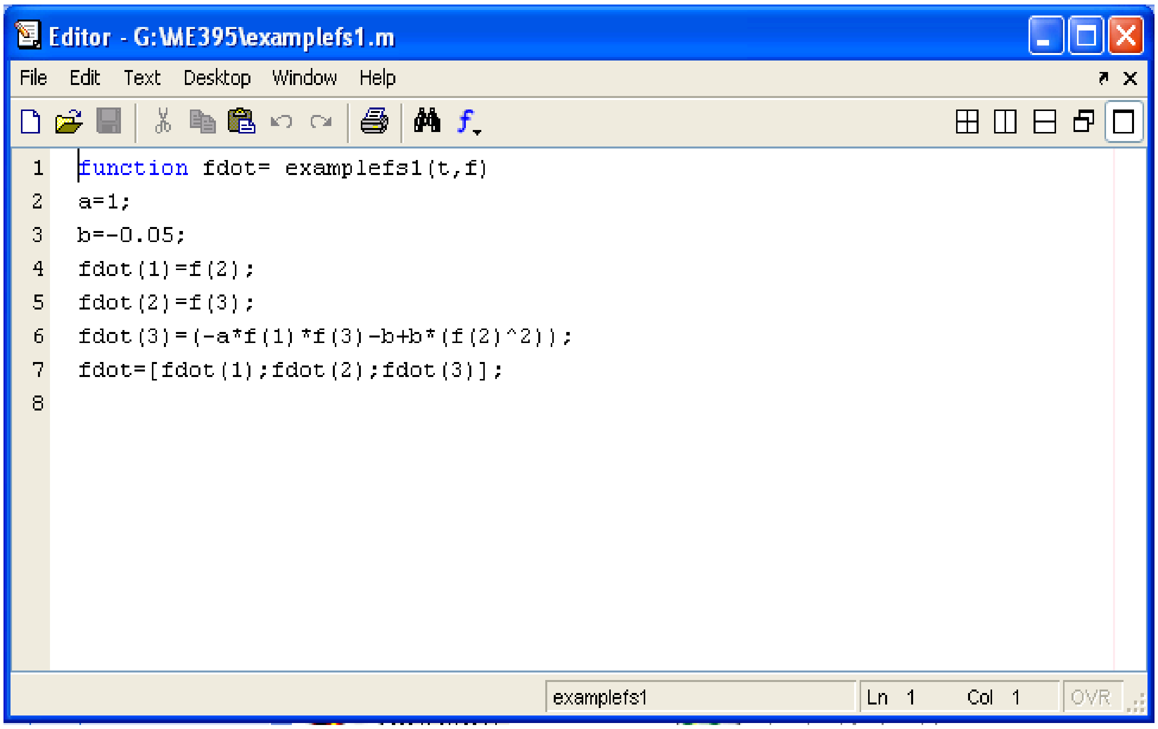
For the future and in order to continue this research, it is suggested that more solutions for the Falkner-Skan wedge and corner flows be generated. In addition, it is also theorized that the Adams number can be related to the pressure gradient that is inherent due to the wedge angle. A suggestion for such research would be to start with an acceleration parameter and using the Bernoulli equation to solve for the pressure gradient as a function of the wedge angle. Perhaps then the expression

$$U_{\infty} = Cx^{m} \tag{42}$$

can be put into a pressure gradient type parameter.

Appendix

Below is screen shot of the mfile that was used in MATLAB that was used for the Falkner-Skan solution for a wedge angle of -0.05.



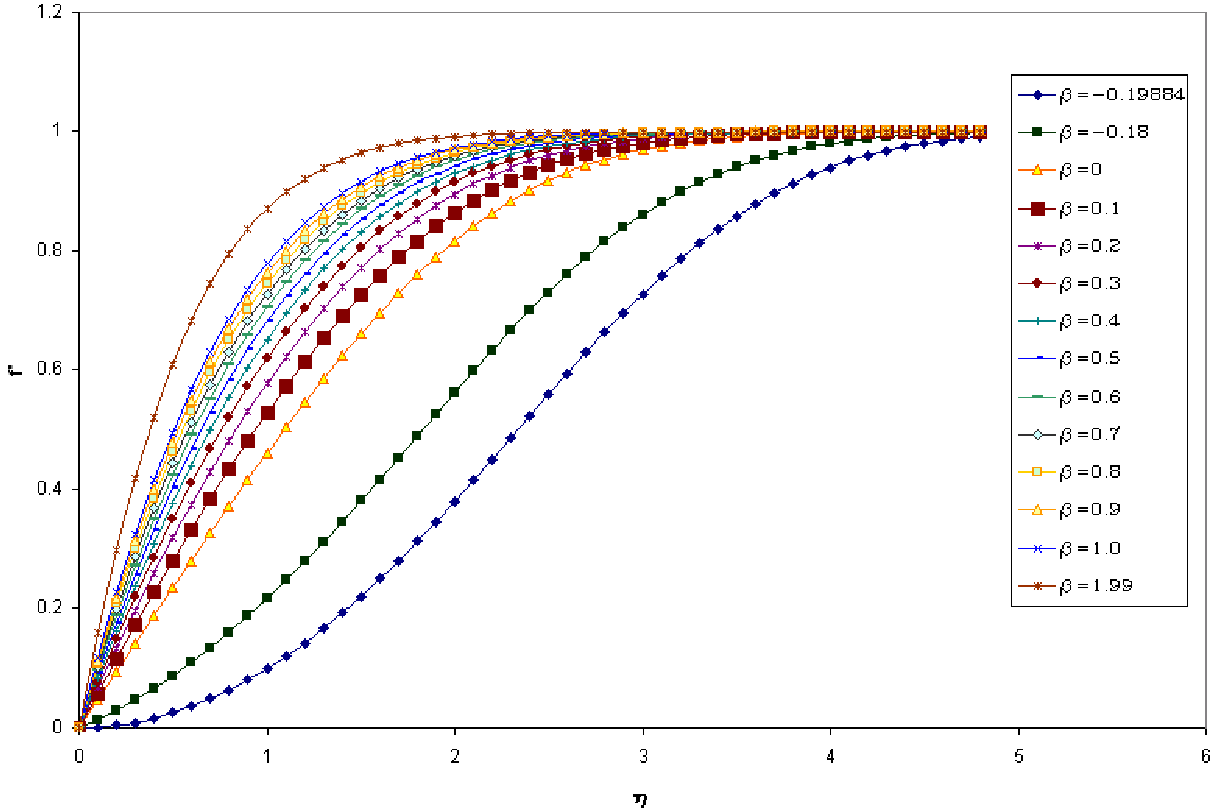


Figure A1. Presented above is $\frac{u}{U_\infty}$ as a function of η, also know as the velocity profile.

								Delta 1		Delta 2
n	f	f'	f"	n^2*f"	n^2*f"/0.74631257	sqrt(2/m+1)	(1-f')	TR	f'(1-f')	TR
0	0	0	0.3192748	0	0	1.449137675	1	0.021922469	0	7.69842E-05
0.022	7.7442E-05	0.007048244	0.321474581	0.000155594	0.00020847	1.449137675	0.992951756	0.021766876	0.006998567	0.000230377
0.044	0.000310477	0.014144872	0.32367304	0.000626631	0.000839581	1.449137675	0.985855128	0.021610218	0.013944794	0.000382595
0.066	0.000700171	0.021289839	0.325868822	0.001419485	0.001901873	1.449137675	0.978710161	0.021452498	0.020836581	0.000533592
0.088	0.001247584	0.02848307	0.328060538	0.002540501	0.003403849	1.449137675	0.97151693	0.021293717	0.027671785	0.00068332
0.11	0.001953771	0.035724457	0.330246766	0.003995986	0.005353957	1.449137675	0.964275543	0.021133879	0.03444822	0.000831731
0.132	0.002819756	0.043013845	0.332426055	0.005792192	0.007760574	1.449137675	0.956986155	0.020972986	0.041163655	0.000978775
0.154	0.003846614	0.050351079	0.334596905	0.0079353	0.010631983	1.449137675	0.949648921	0.020811043	0.047815848	0.001124402
0.176	0.005035415	0.057735966	0.336757778	0.010431409	0.013976354	1.449137675	0.942264034	0.020648053	0.054402524	0.001268563
0.198	0.006387218	0.065168279	0.338907109	0.013286514	0.01780172	1.449137675	0.934831721	0.020484024	0.060921374	0.001411206
0.22	0.007903071	0.072647749	0.341043298	0.016506496	0.02211596	1.449137675	0.927352251	0.02031896	0.067370053	0.001552279
0.242	0.009584012	0.080174069	0.343164713	0.020097098	0.026926771	1.449137675	0.919825931	0.020152869	0.073746188	0.001691729
8.668	7.225320083	1.00000006	-3.46519E-09	-2.60354E-07	-3.48832E-07	1.449137675	-6.04724E-08	-1.26378E-09	-6.04724E-08	-1.26378E-09
8.69	7.247320085	1.000000054	4.1367E-08	3.12388E-06	4.18548E-06	1.449137675	-5.44166E-08	-1.13031E-09	-5.44166E-08	-1.13031E-09
8.712	7.269320087	1.000000048	8.63684E-08	6.55527E-06	8.78297E-06	1.449137675	-4.83391E-08	-1.00366E-09	-4.83391E-08	-1.00366E-09
8.734	7.29132009	1.000000043	1.26635E-07	9.66004E-06	1.29428E-05	1.449137675	-4.29024E-08	-8.99534E-10	-4.29024E-08	-8.99534E-10
8.756	7.313320091	1.000000039	1.56484E-07	1.19973E-05	1.60743E-05	1.449137675	-3.88735E-08	-8.35968E-10	-3.88735E-08	-8.35968E-10
8.778	7.335320093	1.000000037	1.69459E-07	1.30574E-05	1.74947E-05	1.449137675	-3.71236E-08	-8.33269E-10	-3.71236E-08	-8.33269E-10
8.8	7.357320094	1.000000039	1.58324E-07	1.22606E-05	1.64272E-05	1.449137675	-3.86282E-08	#VALUE!	-3.86282E-08	#VALUE!
							sum =	1.442692783	sum =	0.515038198
							Delta 1 =	2.090660465	Delta 2 =	0.746361257

Figure A2. Example of a spread sheet that was used to make some of the necessary calculations. Notice that there truly were 400 steps; therefore, this is only a sample. This particular excel spreadsheet corresponds with a wedge angle of -0.1.

Acknowledgments

The author would like to thank the following people for their help with this research and their overall support. Dr. Jim Smart for teaching fluid mechanics and originally inspiring my interest in the energy sciences as a whole, Dr. John Baker for his help with MATLAB, without which, none of this research could have been completed in a timely fashion, and Dr. Vincent Capece for overseeing all of this research and showing the important aspects of fluid dynamics at a closer level. In addition, I would like to thank all of my classmates whose continued help and support made this possible.

References

Evans, H. L., *Laminar Boundary-Layer Theory*, 1st ed, Addison-Wesley Publishing Company, Reading, 1969, Chaps. 2 and 3

Langtry, R. B., Menter, F. R., "Transition Modeling for General CFD Applications in Aeronautics," *AIAA Journal 2005-522.*

Palm III, W. J., *Introduction to MATLAB 7 for Engineers,* 1st ed, McGraw-Hill Book Company, New York, 1979, Chaps. 3 and 8 .

Schlichting, H., *Boundary-Layer Theory*, 7th ed., McGraw-Hill Book Company, New York, 1979, Chaps. 3 through 9.

Schlichting, H., *Boundary-Layer Theory*, 8th ed., McGraw-Hill Book Company, New York, 2000, Chaps. 5 through 8.